GCE AS MARKING SCHEME

SUMMER 2019

AS (NEW)
FURTHER MATHEMATICS
UNIT 2 FURTHER STATISTICS A
2305U20-1

INTRODUCTION

This marking scheme was used by WJEC for the 2019 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

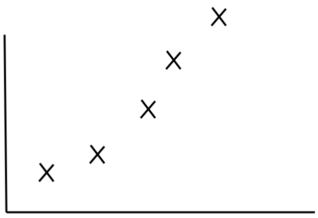
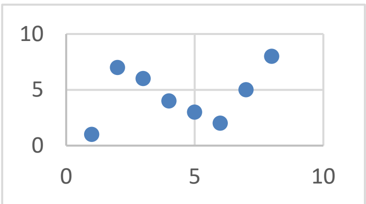
It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

GCE FURTHER MATHEMATICS

AS UNIT 2 FURTHER STATISTICS A

SUMMER 2019 MARK SCHEME

Qu. No.	Solution	Mark	Notes
1(a)		B2	B2 Increasing ranks and non linear. At least 3 points. B1 for positive correlation.
(b)	$\sum d^2 = 58$	B1	
	$r_s = 1 - \frac{6 \times 58}{8 \times 63}$	M1	
	$= 0.3095\ldots = \frac{13}{42}$	A1	
(c)	 		
	Valid comment on agreement. e.g. Both judges agree on the best and worst cheese. e.g. The judges agree on 3 of the 8 cheeses.	E1	
	Valid comment on disagreement. e.g But they almost completely disagree about the others.	E1	
		Total [7]	

Qu. No.	Solution	Mark	Notes
2 (a)	X~B(5, p) Y~B(8, p) Use of $E(XY) = E(X) \times E(Y)$ E(XY) = 5p × 8p $40p^2 = 6.4$ p = 0.4	B1 M1 A1	Si cao
(b)	$Var(X) = 5 \times 0.4 \times 0.6$ $Var(Y) = 8 \times 0.4 \times 0.6$ $Var(X) = 1.2$ $Var(Y) = 1.92$ $E(X^2) = (0.4 \times 5)^2 + 1.2$ $= 5.2$ $E(Y^2) = (0.4 \times 8)^2 + 1.92$ $E(Y^2) = 12.16$ Use of $Var(XY) = E(X^2)E(Y^2) - (E(X)E(Y))^2$ $= 5.2 \times 12.16 - 6.4^2$ $= 22.272$	 B1 M1 A1 (M1) A1 M1 A1 **Total [9]**	 Both si. FT their p M1 FT their p. May be awarded for $E(Y^2)$ FT Their 5.2, 12.16 cao

Qu. No.	Solution	Mark	Notes
3	(Let the random variable X be the number of claims made to the home insurance department in two days.)		
(a)	X~Po(8)	B1	si
	P(X > 11) = 1 – P(X ≤ 11)	M1	si
	= 0.112	A1	0.1119 from calculator or tables.
(b)	(Let the random variable Y be the number of claims made to the pet insurance department in one day.) P(Y=2) = 3×P(Y=4)		
	$\frac{\lambda^2 e^{-\lambda}}{2!} = 3\times\frac{\lambda^4 e^{-\lambda}}{4!}$	M1	
	$24\lambda^2 = 6\lambda^4$	A1	
	$\lambda = 2$	A1	A1 Must reject -2 and 0 if listed as solutions. SC1 for $3 \times P(Y=2) = P(Y=4)$ with subsequent correct working leading to $\lambda = 6$
(c)	P(T > 12) = 1 - P(T < 12)	M1	
	$= 1 - \int_0^{12} \frac{1}{10} e^{-\frac{t}{10}}\, dt$ OR $\int_{12}^{\infty} \frac{1}{10} e^{-\frac{t}{10}} dt$	m1	
	$= 1 - \left[-e^{-\frac{t}{10}}\right]_0^{12}$ $\quad \left[-e^{-\frac{t}{10}}\right]_{12}^{\infty}$		
	P(T > 12) = $e^{\frac{-12}{10}}$		
	=0.301(1942119)	A1	
		Total [9]	

Qu. No.	Solution		Mark	Notes
4(a)		Alternative solution		
	$\int_0^1 kx\,dx + \int_1^2 kx^3\,dx = 1$	$\int_0^1 kx\,dx + \int_1^x kx^3\,dx$	M1	M1 limits and =1 not required for this mark.
	$\left[\frac{kx^2}{2}\right]_0^1 + \left[\frac{kx^4}{4}\right]_1^2 = 1$	$F(x) = \left[\frac{kx^2}{2}\right]_0^1 + \left[\frac{kx^4}{4}\right]_1^x \quad (1 \le x \le 2)$	A1	
	$\frac{k}{2} + \left(4k - \frac{k}{4}\right) = 1$	$F(x) = \frac{k}{2} + \left(\frac{kx^4}{4} - \frac{k}{4}\right)$ and $F(2) = 1$	m1	Substituting in limits.
	$\frac{17k}{4} = 1$	$\frac{k}{4} + \frac{16k}{4} = 1$		
	$k = \frac{4}{17}$	$k = \frac{4}{17}$	A1cao ***ag**	A1 Convincing with at least one step shown between m1 and answer.
(b)	$E(X) = \int_0^1 kx^2\,dx + \int_1^2 kx^4\,dx$		M1	Attempt at $\int xf(x)dx$ with at least one power increasing
	$= \left[\frac{kx^3}{3}\right]_0^1 + \left[\frac{kx^5}{5}\right]_1^2$		A1	
	$= \frac{k}{3} + \left(\frac{32k}{5} - \frac{k}{5}\right)$		m1	Substituting in limits.
	$= \frac{392}{255} = 1.54\ (3sf)$		A1	$1.537254\ldots$ Correct answer with no working scores zero marks.
(c)	$E(3X-1) = 3E(X) - 1$		M1	
	$= \frac{307}{85} = 3.61$		A1	FT their $E(X)$
	$E(X^2) = \int_0^1 kx^3\,dx + \int_1^2 kx^5\,dx$		M1	Attempt at $\int x^2 f(x)dx$ with at least one power increasing
	$Var(X) = \left[\frac{kx^4}{4}\right]_0^1 + \left[\frac{kx^6}{6}\right]_1^2 - \left(\frac{392}{255}\right)^2$		M1	M1 subtracting their $(E(X))^2$
	Awrt 0.166		A1	FT their $E(X)$ and their $E(X^2)$ dep M1M1 and $Var(X)$ is positive.
	Var(3X-1) = 9Var(x) =1.496		M1 A1	A1 cao
			Total [15]	

<table>
<tr><th>Qu. No.</th><th>Solution</th><th>Mark</th><th>Notes</th></tr>
<tr><td>5(a)</td><td>H_0: Birth months are evenly distributed across the year.</td><td>B1</td><td></td></tr>
<tr><td></td><td>Valid conclusion
e.g. There is a bias for NHL players to have birthdays earlier in the year.
e.g. The large χ^2 value comes from a greater number than expected number of births for Jan – Mar and fewer than expected births for Oct-Dec.</td><td>B1</td><td></td></tr>
<tr><td></td><td>The p value is much smaller than 1%</td><td>B1</td><td>Allow less than 5% or 0.05 Accept CV method.</td></tr>
<tr><td>(b)</td><td>H_0: The data can be modelled by the uniform distribution.
H_1: The data cannot be modelled by the uniform distribution.</td><td>B1</td><td>Both</td></tr>
<tr><td></td><td>Expected frequencies are
<table>
<tr><td>Jan</td><td>Feb</td><td>Mar</td><td>Apr</td><td>May</td><td>Jun</td><td>Jul</td><td>Aug</td><td>Sep</td><td>Oct</td><td>Nov</td><td>Dec</td></tr>
<tr><td>6.25</td><td>6.25</td><td>6.25</td><td>6.25</td><td>6.25</td><td>6.25</td><td>6.25</td><td>6.25</td><td>6.25</td><td>6.25</td><td>6.25</td><td>6.25</td></tr>
</table></td><td>B1</td><td>B1 for 6.25 AND no combined classes.</td></tr>
<tr><td></td><td>Use of $\chi^2 = \sum \frac{(O-E)^2}{E}$ OR $\chi^2 = \sum \frac{O^2}{E} - N$
$= \frac{(3-6.25)^2}{6.25} + \frac{(7-6.25)^2}{6.25} + \cdots + \frac{(5-6.25)^2}{6.25} + \frac{(6-6.25)^2}{6.25}$</td><td>M1</td><td>Does not need full table.
$\frac{3^2}{6.25} + \frac{7^2}{6.25} + \cdots + \frac{6^2}{6.25} - 75$</td></tr>
<tr><td></td><td>=15.4</td><td>A1</td><td></td></tr>
<tr><td></td><td>DF = 11</td><td>B1</td><td>si</td></tr>
<tr><td></td><td>10% Crit val = 17.275</td><td>B1</td><td>FT their dof</td></tr>
<tr><td></td><td>Since 15.4 < 17.275 Do not reject H_0.</td><td>B1</td><td>FT their CV and χ^2</td></tr>
<tr><td></td><td>We can conclude that the data can be modelled by the uniform distribution.</td><td>B1</td><td>Only award final B1 if previous 3 B1 awarded.</td></tr>
<tr><td></td><td></td><td>Total [11]</td><td></td></tr>
</table>

Qu. No.	Solution	Mark	Notes
6(a)	Valid comment. Eg. Data suggests a non-linear relationship. The scatter diagram seems to have three distinct sections. The regression line will give an underestimate for some values of x e.g. between 2500 and 4000 and an overestimate for others e.g. between 1500 and 2500. Three separate lines for the distinct sections would be better.	E1	
(b)	$b = \frac{348512820.6}{2869673.03}$	M1	
	$b = 121.4468746$	A1	A1 121.4 or 121.45 or better
	$a = \frac{3907142}{37} - 121.4468746 \times \frac{93160}{37}$	M1	
	$a = -200185.1037$	A1	A1 awrt -200000
	y = -200185 + 121.4x	A1	A1 FT 'their' gradient and intercept dep on at least one M1 awarded.
		Total [6]	

Qu. No.	Solution	Mark	Notes
7(a)	$A = \dfrac{115 \times 115}{379} = 34.8945$	M1 A1	oe method
	$B = \frac{(51-57.9551)^2}{57.9551}$ $C = \frac{(6-16.6887)^2}{16.6887}$	M1	M1for either method M1A0 for one correct χ^2 contribution.
	B = 0.83467 C = 6.84585	A1	Both
(b)	H_0: Site of injury and sport are independent. H_1: Site of injury and sport are not independent.	B1	OR H_0: There is no association between site of injury and sport. H_1: There is an association between site of injury and sport.
	Degrees of freedom = (7 – 1)×(3 – 1) = 12	B1	si
	5% critical value = 21.026	B1	FT their dof
	Since 116.16 > 21.026 Reject H_0.	B1	FT their CV
	There is sufficient evidence to suggest that the site of injury and the sport are not independent.	B1	Only award final B1 if previous 3 B1 awarded.
(c)	hand/fingers. (Due to the high values in Football and Basketball.)	E1	
	This is not surprising to see disproportionally fewer injuries to the hand / fingers in football and disproportionally more injuries to the hand / fingers in basketball.	E1	Must convey the idea of less use of fingers in football and/or more in basketball.
(d)	The test also depends on the degrees of freedom, which are different in this case so we cannot compare the two totals for the χ^2 contributions. She should compare p-values instead.	E1 E1 **Total [13]**	Must convey the idea of an unfair comparison because the dof are different.